**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

X = 600

Meam = 45+10 = 55

Std. Deviation = 8

from scipy import stats

round(1-stats.norm.cdf(60,loc=55,scale=8),5)

0.26599

Option B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

False, since mean = 38 and standard deviation = 6, that means most of the ages are lying between 32 and 44.

Z score for 44

from scipy import stats

round(1-stats.norm.cdf(44,loc=38,scale=6),5)

0.15866 = 63 people out of 400

Z score between 38 and 44

from scipy import stats

stats.norm.cdf(44,loc=38,scale=6)-stats.norm.cdf(38,loc=38,scale=6)

0.3413447460685429 = 137 people out of 400

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

True

from scipy import stats

stats.norm.cdf(30,loc=38,scale=6)

0.09121121972586788

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans- As we know that if X <" N(µ1, Ã 1^2 ), and Y <" N(µ2, Ã 2^2 ) are two independent random variables then X + Y <" N(µ1 + µ2, Ã 1^2 + Ã 2^2 ) , and X " Y <" N(µ1 " µ2, Ã 1^2 + Ã 2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z <" N(aµ1 + bµ2, a^2Ã 1^2 + b^2Ã 2^2 ).

Therefore in the question

2X1~ N(2 u,4 Ã ^2) and

X1+X2 ~ N(µ + µ, Ã ^2 + Ã ^2 ) ~ N(2 u, 2Ã ^2 )

2X1-(X1+X2) = N( 4µ,6 Ã ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Since it is 99% we need to add 0.5% on either side, i.e 0.005 = 0.99+0.005 = 0.995

Now Z value of 0.005 = from scipy import stats

stats.norm.ppf(0.005)

-2.575829303548901

Now Z value of 0.995 = from scipy import stats

stats.norm.ppf(0.995)

2.5758293035489004

Z = (x-Mean)/std.D , x = std.D\*Z + Mean , x = 20\*Z + 100

A= (20 \* (-2.57)) + 100 = 48.5

B = (20\* 2.57) + 100 = 151.5 option D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

import numpy as np

from scipy import stats

from scipy.stats import norm

Mean = 5+7

print('Mean Profit is Rs', Mean\*45,'Million')

SD = np.sqrt((9)+(16))

print('Standard Deviation is Rs', SD\*45, 'Million')

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

Mean Profit is Rs 540 Million

Standard Deviation is Rs 225.0 Million

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

From Z table we know that 5th percentile value is -1.645

X = std.D\*Z + Mean = import numpy as np

X= 540+(-1.645)\*(225)

print('5th percentile of profit (in Million Rupees) is',np.round(X,))

5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

Making loss i.e X<0

Division 1

stats.norm.cdf(0,5,3)

0.0477903522728147

Division 2

stats.norm.cdf(0,7,4)

0.040059156863817086

Division 2 will face more loss